a.s.]. If $(X, \mathfrak{T})$ is admissible, $M(\mathfrak{T}) = R^d$, and each T_i is a.s. differentiable, then the Oseledec theorem implies that $\|\mathbf{D}_X(T_n \cdots T_1)\mathbf{v}\|$ has a.s. an exponential growth rate equal to one of at most d values [where $\mathbf{D}_X(T_n \cdots T_1)$ is the $d \times d$ derivative matrix of $(T_n \cdots T_1)$ evaluated at X], provided $E \log^+\|\mathbf{D}_X(T_1)\| < \infty$. [Take $\mathbf{A}_i$ to be the derivative matrix for T_i evaluated at $T^{(i-1)}(X)$, and apply the chain rule.] Thus one can assign Lyapunov exponents to an admissible pair $(X, \mathfrak{T})$ in this context. It is natural to guess that this result extends to pairs $(X, \mathfrak{T})$, for which $M(\mathfrak{T})$ is a compact d-dimensional Riemannian manifold and each transformation in $\mathfrak{T}$ is a.s. differentiable. Kifer proves this in Chapter III. (He states his result for vector bundles.)

Chapter IV and the first part of Chapter V give partial answers to the following questions: When do the Lyapunov exponents coincide? When is the largest one positive? When is the largest one continuous with respect to the distribution of $(X, \mathfrak{T})$? The last half of Chapter V applies the preceding theory to an analysis of the asymptotic behavior of solutions to stochastic differential equations.

Despite some drawbacks (too many technicalities in some places, too few applications, no index, pages without chapter headings), this book should be read by anyone with a solid background in analysis and probability who wishes to learn about the ergodic theory of random transformations. A concurrent reading of the works by Bougerol and Lacroix (1985) and Cohen et al. (1986) on random matrices is helpful. I hope that Kifer's work will stimulate more research in this interesting area.

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Asymptotic Methods in Statistical Decision Theory.

Lucien LeCam. New York: Springer-Verlag, 1986. xxvi + 742 pp. $49.95.

For more than 30 years Lucien LeCam has been actively constructing his own view of asymptotic statistical theory. This massive book is the only comprehensive account of the entire theory developed by LeCam and his disciples. It is an important but difficult book. I will attempt to mention the principal contents of the book and also help ease the reader's path.

LeCam's starting point is a reformulation of Wald's decision theory. He first presented this reformulation in LeCam (1955), further developed and refined it in succeeding papers, and gives it here in Chapters 1–3, 5, 7, and 8. These chapters also contain important special material required for the later asymptotic theory. This material, particularly that concerning the related concepts of deficiency and insufficiency, forms an important bridge to the asymptotics.

Chapter 6 introduces the notion of contiguity, one of LeCam's best-known and most important contributions to asymptotic theory. This chapter is largely independent of the decision-theoretic chapters surrounding it.

Chapters 10–12 are the climax of the first half of the book. Chapter 11 alone occupies more than 100 pages. These chapters establish extremely general conditions under which a sequence (or even a net!) of experiments suitably converges to one involving only normal distributions. If so, statistical procedures that behave well in the limiting normal problem will do asymptotically as well in the original sequence. Chapter 10 discusses the local theory in (shrinking) neighborhoods about a specified parameter point. Chapter 11 presents global theories yielding approximations that can be used over the whole parameter space. Chapter 12 describes the existence of approximately Gaussian posterior distributions and the asymptotic behavior of Bayes procedures.

Chapters 16 and 17 occupy another 175 pages. They return to issues raised in Chapters 10–12, but in settings that, although still general, are specific enough to yield moderately explicit statements about asymptotic risks, precise bounds on rates of convergence, and related matters. These last two chapters are really a redevelopment of asymptotics rather than a direct continuation of the earlier chapters; only a few technical results are needed here from Chapters 10–12, along with a general central limit theorem for infinitely divisible distributions (proved in Chap. 15).

The book places severe demands on the reader's mathematical and statistical background. For mathematical background, a solid basic education in functional analysis is particularly important. In an appendix LeCam surveys the functional analysis used, squashing two full graduate courses in functional analysis into just 70 pages. Of course, if one is willing to skip a few proofs one can get by with less than the two courses surveyed here, but a first course in functional analysis seems essential.

For statistical background, LeCam suggests that the reader should have been "exposed" to Bickel and Doksum (1977) or Ferguson (1967). LeCam should also have mentioned a book [such as that by Scheffé (1959)] that contains important background material for Chapter 11. But I think an entirely different sort of statistical background is also necessary: One needs to be familiar with LeCam's achievements as further developed and interpreted by others. The following paragraphs mention some examples.

Part (b) of Theorem 1 on pages 16–17 is sometimes called the Stein–LeCam theorem. It is a general, abstract version of Stein (1955), but with an entirely different proof. It contains a formal statement of Blyth's (1951) method for proving admissibility. That method is, of course, a cornerstone for many admissibility proofs in the decision-theoretic literature, such as Stein (1959) or Brown and Hwang (1982). Farrell (1966) included a statement and proof of a less general form of the Stein–LeCam theorem, written in a style familiar to most statistically trained readers (see also Brown 1987, pp. 254–268). It is not necessary to have read any of these works to comprehend the version in LeCam's book, but they add layers of meaning to the theorem that are not hinted at in the book.

According to the introduction, Theorem 1 on page 115 is a "fairly general form . . . of what is sometimes called the Hájek–LeCam asymptotic minimax theorem" (p. xviii). I doubt that many readers will recognize it as such—it appears to be a result about the approximability of general decision procedures by nonrandomized ones. (Perhaps LeCam even meant to refer to the earlier Theorem 1 on p. 109.) To get some idea of the nature of the "Hájek–LeCam" theorem and its role in statistics, read Lehmann (1983, chap. 6) and then Hájek (1972).

In what is essentially a digression LeCam gives an elegant proof of the Hunt–Stein theorem in Theorem 1 (p. 152). [I believe the following hypothesis should replace the assumption concerning T_0: Let $r_0 : \theta \to r_0(\theta)$ be invariant under γ, with $r_0(\theta) \geq r(\theta, T_0)$. The conclusion should then be modified to be $r(\theta, T) \leq r_0$.] Lehmann (1986, chap. 9) and Kiefer (1966, pp. 261–265) provided reasonable preparation for this version of the Hunt–Stein theorem. Brown (1986) included a treatment similar to LeCam's, but in a more conventional statistical setting, and thus may also be useful.

Contiguity plays a key role in the asymptotic normality result (Theorem 1, p. 192) already mentioned in connection with the Hájek–LeCam theorem. Nevertheless, contiguity has other important applications. Hájek and Šidák (1967) gave a good idea of some of these further uses; their work is thus valuable preparation for Chapter 11; Chapter 6, Section 3; Chapter 10, Section 5; and Chapter 16, Sections 2 and 3. For example, compare Hájek and Šidák's pages 204–210 with LeCam's Proposition 2 (Chap. 16, p. 470). Roussas (1972) is an alternate reference on contiguity.

Sections 4 and 5 of Chapter 16 concern the construction of estimators that in standard settings are $\sqrt{n}$-consistent. Familiarity with Wald's (1949) proof of the ordinary consistency of the maximum likelihood estimator is a useful prerequisite for this material even though it is neither used nor cited here. More directly relevant is the need for an example illustrating the general techniques described. Birge (1987a,b) contains such an example, providing striking evidence of the potential power of the results in these sections. [As LeCam notes, some of his results are due originally to Birge (1980).]

Toward the end of Chapter 17 (Sec. 6) LeCam sketches robust asymptotic minimax results of Millar (1979) and Beran (1981) in the context of this chapter. Perhaps the reader will find the original papers of Millar and Beran to be more easily managed. These papers would be useful preparation not only for this chapter, but for Chapter 11 as well. Wang (1986) provided a useful concrete example of what can be obtained from Millar's results. Chapter 17 concerns in part the concept of differentiability in quadratic mean (which is closely related to Frechet differentiability of the family of densities). Hüber (1981) gave a good discussion of this latter concept, as well as further material that could prove useful for understanding LeCam's presentation.

LeCam states results in the maximum possible generality. The level of generality is gradually reduced through the book, but only to the degree necessary to produce further results. In only a few places are more particular results stated because of their convenience or special applicability (e.g., see Chap. 11, Sec. 10, about χ^2 tests). As LeCam says, "The presentation proceeds from the general to the particular since this seemed the best way to emphasize the basic concepts" (p. xiii). Surely this orientation has been an essential ingredient in the important innovations (such as contiguity and differentiability in quadratic mean) LeCam has brought to asymptotic theory. It also might seem to guarantee that the results obtained should have the widest possible applicability.

On the other hand, this orientation and its execution place heavy burdens on the reader. For example, Theorem 1 of Chapter 11 (Sec. 8) is a basic result about the existence of Gaussian-shift experiments asymptotically equivalent to a given sequence of experiments. The theorem applies if "the requirements (1) to (4) above" are satisfied, in addition to "the requirements (A), (B), (D), (E), and (FF) of Section 6" (p. 284). [Actually, requirements (B), (D), and (E) are formally defined in Sec. 5.] These requirements are not of the classical, easily understood, quickly checked variety, such as "$f_\theta(x)$ is continuous in θ for almost all x." Rather, as an example, requirement (A) says that "for every favorable θ every $s \in \theta$ such that $q(\theta, s)$ remains bounded [as $n \to \infty$] is accessible" (p. 247). (The terms "favorable," "q," and "accessible" were also defined in Sec. 6.)

In short, the organization and execution of the book require that the reader carry a large bulk of technical material over long distances, and nearly the entire book must be carefully read at least once through to discern the full, correct, logical route to specific applications.

Given the nature of this book, a good index would be a valuable tool. The present index is helpful but incomplete: It only lists definitions, and this cannot help the reader discover that "good," defined on page 284, is discussed on page 610, and so forth.

The bibliography is complete in one respect and distressingly inadequate in others. It contains a full, useful compilation of works dealing with asymptotics from the perspective of the book (e.g., those by LeCam himself, and by Hájek, Torgerson, Beran, Millar, Birge, and Moussatat). On the other hand, references to useful background material (a very small portion of which has been cited in this review) are meager. References to the many specific applications of asymptotic theory are almost entirely absent.

Finally, LeCam cites many important works of other authors by mentioning the name of the author, but he fails to provide any further information or citation in the text or bibliography. Here are the citations for a few of the nearly two dozen instances I found: Birnbaum (1955) [mentioned on pp. 196, 297; LeCam should also mention Stein (1956a) in this context], Neyman (1959) or Neyman and Scott (1967) (mentioned on p. 207), Stein (1956b) (mentioned on pp. 235, 248, 256), Stone (1975) (mentioned on p. 235), Chernoff and Lehmann (1954) (mentioned on p. 306), and Ferguson (1973) (mentioned on p. 618).

In conclusion, this is a book for serious study. The mathematically or statistically unprepared reader or the prepared but casual reader will likely get nothing from it but a headache. But the prepared and diligent reader will find a gold mine, from which can be distilled an effective and powerful understanding of statistical asymptotics.

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Stochastic Geometry and Its Applications.

D. Stoyan, W. S. Kendall, and J. Mecke. Chichester, U.K.: John Wiley, 1987. 345 pp. $45.59.

Geometric probability deals with questions such as the following: For three points picked randomly in a region, what is the chance they form the vertices of an acute triangle? If you throw discs randomly onto a square, what is the chance the square is completely covered? For the planar tessallation (pattern of regions) induced by randomly placing lines, what is the area of a typical polygon? The name *stochastic geometry* has been given a slight nuance: the study of probability models for spatial geometric patterns, with a view toward statistical applications. I shall treat separately the book's primary aim of describing probability models and their properties, and its secondary statistical applications.

The style is a little unusual, using rigorous and abstract mathematical language but omitting most of the proofs. A lot of material is covered in 345 pages, but the reader needs some mathematical sophistication, approximately at the level of a graduate course in probability theory. There are three intertwined themes. One treats spatial point processes, starting with the Poisson process and its variations and continuing to clustering models, hard-core models, and Gibbs models, and including a general discussion of moment measures, Palm (i.e., conditional) distributions, and abstract point processes. The second repeats the first in the context of random sets, starting from the simplest Boolean model (iid sets with Poisson centers). The third is a collection of special topics: Poisson processes of lines, the induced tessallation, the Voronoi tessallation, a chapter on stereology (studying three-dimensional objects when one has only data for slices), fiber processes of line segments (analogous to earthquake fault lines), and random triangles via sophisticated parameterization of the "shape-space" of triangles.